

Trend and long-run relations in electricity prices

Why pre-filtering is inevitable

Matteo Pelagatti²

jointly with

A. Gianfreda¹ L. Parisio² P. Maranzano²

¹Free University of Bozen

²University of Milano - Bicocca

Third AIEE Energy Symposium
Milan, 10-12 December 2018

Motivation

Google Scholar

+mean +reversion +electricity +price



Articles

About 13,100 results (0.04 sec)

Any time

Since 2018

Since 2017

Since 2014

Custom range...

Sort by relevance

Sort by date

- include patents
- include citations

Create alert

[HTML] Regime jumps in **electricity** prices

[R Huisman](#), R Mahieu - *Energy economics*, 2003 - Elsevier

... This highlights the importance of **mean-reversion** in **electricity price** processes ... Standard errors are between parentheses. Note that the inclusion of **mean-reversion** leads to a richer specification of the **electricity price** process indicated by the lower log-likelihood values ...

☆ Cited by 419 Related articles All 22 versions Web of Science: 142

[BOOK] Stochastic models of energy commodity prices and their applications: **Mean-reversion** with jumps and spikes

[S Deng](#) - 2000 - ei.haas.berkeley.edu

... congestion on key transmission lines. Within a couple of days the **price** fell back to the ... Figure 3: Generation Stack for **Electricity** in a Region Although the **mean reversion** is well studied, there has been little work examining the ...

☆ Cited by 462 Related articles All 9 versions

Motivation

- Many authors list **mean reversion** as an important feature of electricity prices.
- In most electricity markets **gas, coal and oil prices** are **important drivers** of electricity prices.
- There is virtual unanimity in holding **gas, coal and oil log-price dynamics** as well approximated by **integrated processes** (mostly as random walks).
- Therefore, mean-reversion of electricity prices does not seem plausible for most markets.

Why do researchers find/assume mean reversion?

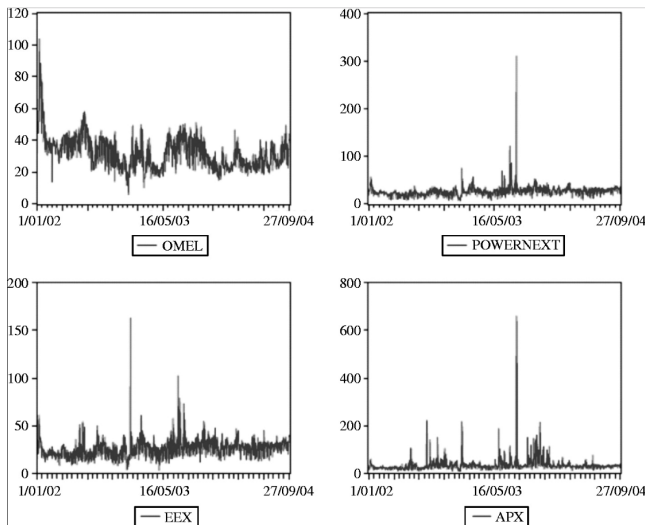
I take da Silva, Soares (2007) as one of many examples.

According to both unit root tests, the hypothesis of stationarity cannot be rejected for the four series. Hence, although very appealing, co-integration does not seem the appropriate method to use in the analysis; as such an analysis would always find prices to be co-integrated.

	OMEL	POWERNEXT	APX	EEX
ADF	-5.401	-10.540	-10.512	-10.560
PP	-9.760	-22.585	-17.639	-18.183

Notes: 1 per cent Critical value (MacKinnon critical values for rejection of hypothesis of a unit root), -3.4396; 5 per cent Critical value, -2.8648; 10 per cent Critical value, -2.5685

Can you use least-squares based methods on these data?



The drivers of electricity prices

- Electricity prices are determined in the long-run mainly by demand level, fuel prices and the mix of generation technologies.
- Hourly and daily prices are buried into high-variance leptokurtic noise produced by many factors:
 - ▶ line congestions (Italy can be split in up to six zones if this happens),
 - ▶ firms' strategies and, possibly, exercise of market power at particular hours/seasons,
 - ▶ plant maintenance (both programmed and unexpected),
 - ▶ start-up costs of power plants,
 - ▶ intermittent RES generation,
 - ▶ import/export prices effects.

How important is the noise in daily prices?

Consider the Unobserved Component Model

$$\log(p_t) = \mu_t + \gamma_t + \omega_t + \beta^\top \mathbf{x}_t + \varepsilon_t,$$

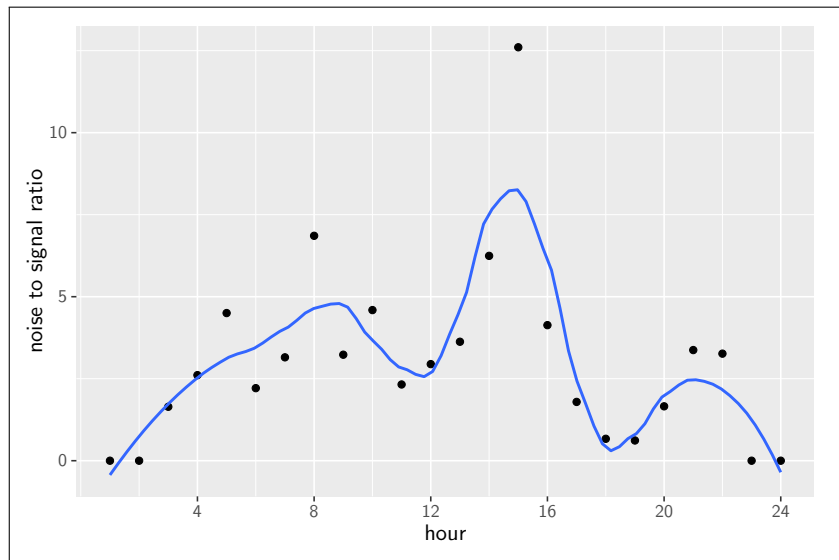
with

- p_t are daily prices of a given hour,
- $\mu_t = \mu_{t-1} + \eta_t$ is a random walk trend component,
- γ_t is a stochastic dummy weakly seasonal component,
- ω_t is a smooth yearly seasonal component made of stochastic sinusoids,
- $\beta^\top \mathbf{x}_t$ are regression effects capturing demand and holiday effects,
- ε_t is the noise component;

and where the noise-to-signal ratio is defined as

$$c = \frac{\text{VAR}(\varepsilon_t)}{\text{VAR}(\eta_t)}.$$

Noise-to-signal ratio in Italian electricity prices



I(1) process observed with noise

Let us consider the simplest I(1) process, the random walk, observed with noise

$$\begin{aligned}y_t &= x_t + \varepsilon_t, & \varepsilon_t &\sim \text{WN}(\sigma_\varepsilon^2) \\x_t &= x_{t-1} + \eta_t, & \eta_t &\sim \text{WN}(\sigma_\eta^2)\end{aligned}$$

This process has the reduced ARIMA(0, 1, 1) form

$$\Delta y_t = \eta_t + \varepsilon_t - \varepsilon_{t-1} = \zeta_t - \theta \zeta_{t-1}, \quad \zeta_t \sim \text{WN}(\sigma^2)$$

with

$$\theta = 1 + \frac{\lambda - \sqrt{\lambda^2 + 4\lambda}}{2}, \quad \sigma^2 = \frac{\sigma_\varepsilon^2}{\theta}.$$

where $\lambda = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ is the signal-to-noise ratio.

I(1) process observed with noise

If the signal-to-noise ratio is low, then the coefficient θ is close to one and the MA operator almost annihilate the difference operator:

$$(1 - L)y_t = (1 - \theta L)\zeta_t,$$

and the I(1) process is hard to distinguish from a white noise.

At the same time, if, as in ADF test, the MA(1) has to be approximated by an AR(p), when θ is close to one, the value of p must be very large to obtain a decent approximation:

$$(1 - \theta L)^{-1} = 1 + \theta L + \theta^2 L^2 + \theta^3 L^3 + \dots$$

Thus, the critical values of the ADF will be bad approximations as well.

The closer θ to zero, the better for ADF and Johansen tests.

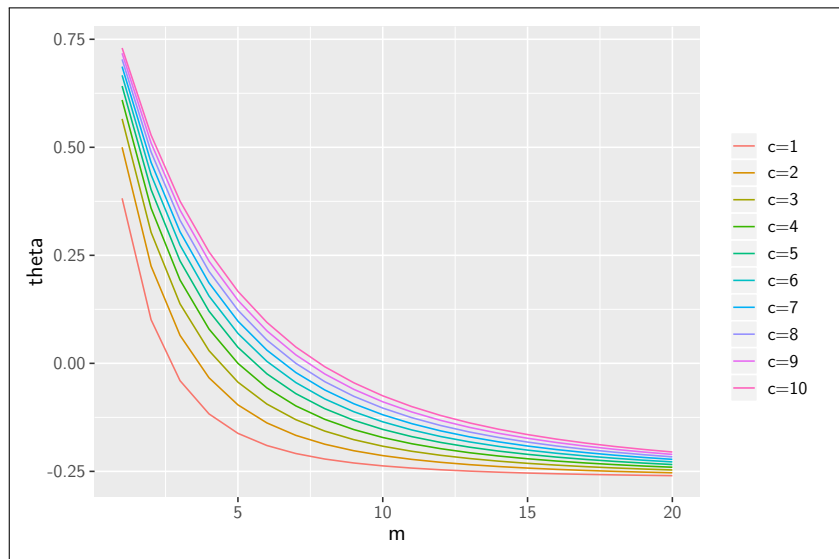
How we dealt with the problem

To study integration and RES effects on some European electricity markets we filtered price time series using one or more of the following approaches.

- Taking **weekly means or medians** (Bosco et al. 2010, Gianfreda et al. 2016).
- Developing **robust integration and cointegration tests** (Bosco et al. 2010, Pelagatti & Sen 2013).
- Extracting the long-run component using UCM and **Kalman filtering/smoothing** (Gianfreda et al. 2016, 2018).

In this talk, we want to show the advantages of using these filtering methods when working on electricity prices.

The mean over m -observations of a RW + WN is IMA(1, 1) with the following θ



Data generating process for the ADF test

The data are generated by summing a random walk

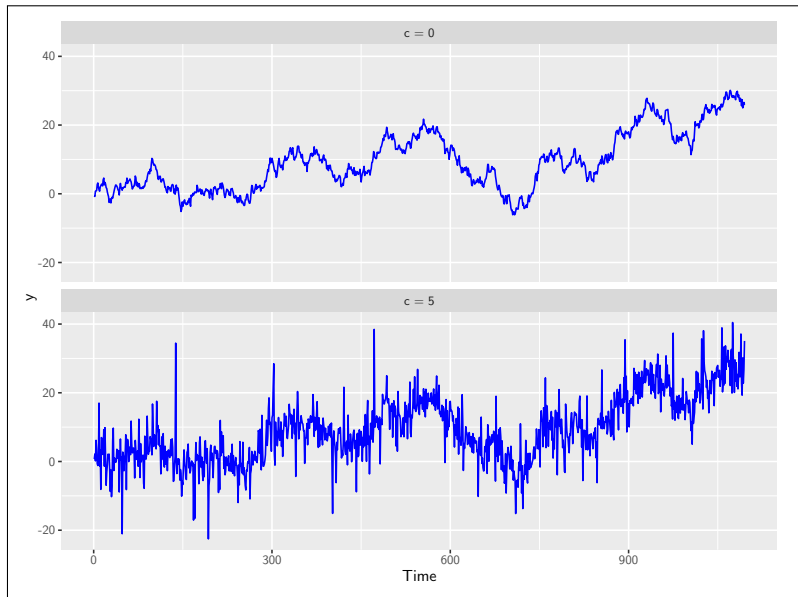
$$x_t = x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0, 1),$$

with a **leptokurtic noise** z_t , generated by a (standardized) Student's t random variable with ν degrees of freedom:

$$y_t = x_t + \sqrt{c}z_t.$$

The parameter c is fixed and represents the **noise-to-signal ratio**.

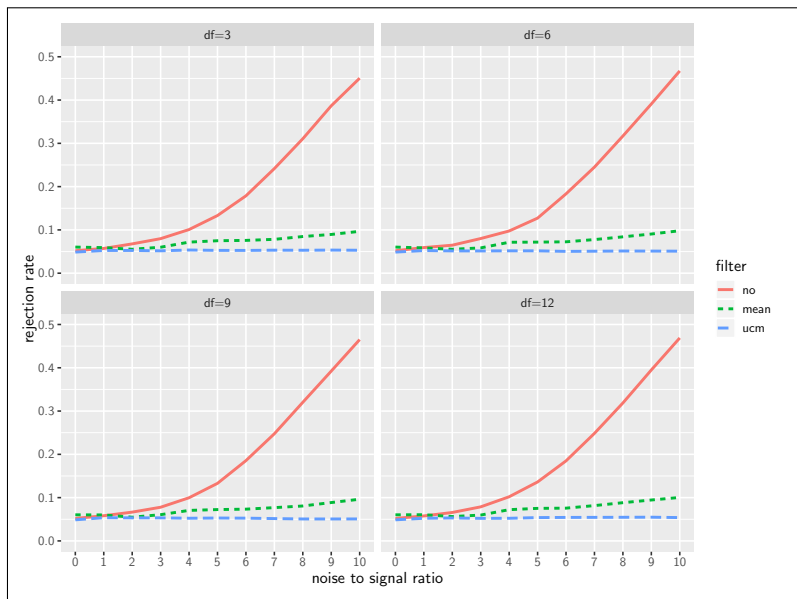
Sample path of RW and RW + NOISE



Simulation scheme

- We simulated 10,000 time series paths of length 1095 (3 years of daily observations) for the following values:
 - noise-to-signal ratio $c = 0, 1, \dots, 10$;
 - Student's t degrees of freedom $df = 3, 6, 9, 12$.
- On each simulated time series we applied the ADF test with the number of lags determined using AIC (max lags = 10).
- We applied the ADF test to:
 - no raw data
 - mean aggregated time series by taking the mean every 7 observations (from daily to weekly)
 - ucm smoothed level in a *random walk plus noise* UCM (variances estimated by Gaussian QML)

Rejection rate of the ADF test



Data generating processes for Johansen's test

We assume that our $k = 4$ time series in \mathbf{y}_t are generated by the following VECMs with $r = 1$ and $r = 2$ cointegrating relations

$$\Delta \mathbf{x}_t = \begin{bmatrix} 0.0 & 0.0 \\ 0.1 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.2 \end{bmatrix} \begin{bmatrix} 1.0 & -0.5 & -0.5 & 0.0 \\ 0.0 & 1.0 & 0.0 & -1.0 \end{bmatrix} \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,$$

$$\Delta \mathbf{x}_t = \begin{bmatrix} 0.0 \\ 0.1 \\ -0.1 \\ 0.1 \end{bmatrix} [1 \quad -1 \quad 1 \quad -1] \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,$$

plus a leptokurtic noise z_{it} , generated by (standardized) Student's t random variables with ν degrees of freedom:

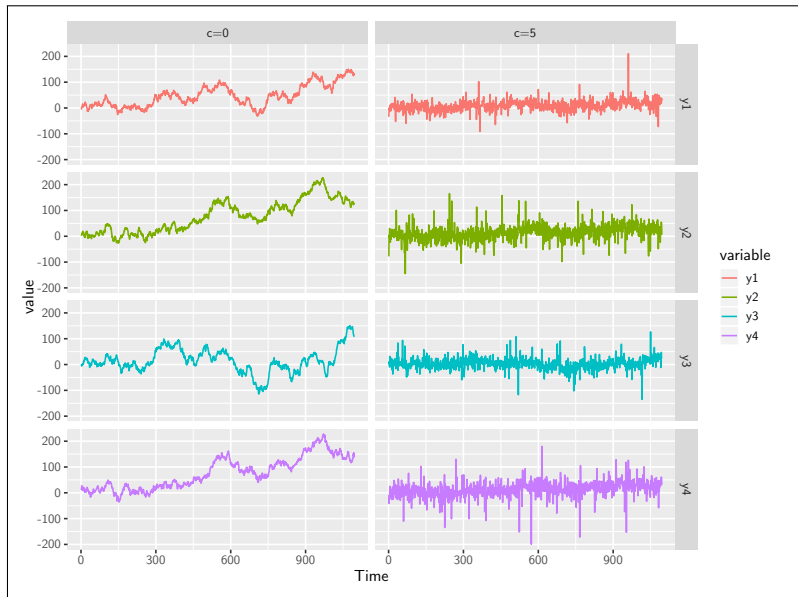
$$y_{it} = x_{it} + \gamma_i z_{it}, \quad \gamma_i^2 = c \frac{\text{VAR}(\Delta y_{it})}{\text{VAR}(z_{it})}.$$

The parameter c is fixed and represents the noise-to-signal ratio.

Simulation scheme

- The characteristic roots of the above model written in VAR(1) form are, respectively: 1.0 1.0 1.0 0.7 and 1.00 1.00 0.85 0.80
- We simulated 10,000 time series paths of length 1095 (3 years of daily observations) for the following values:
 - noise-to-signal ratio $c = 0, 1, 2, \dots, 10$
 - Student's t degrees of freedom $df = 3, 6, 9, 12$
- On each simulated time series quartet, we applied Johansen's trace test.

Sample path of VECM and VECM + NOISE

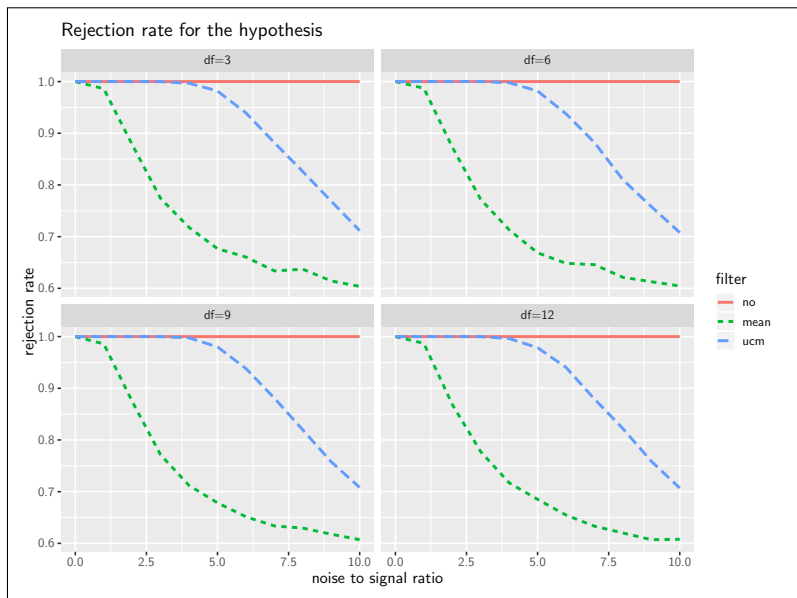


The filters we used

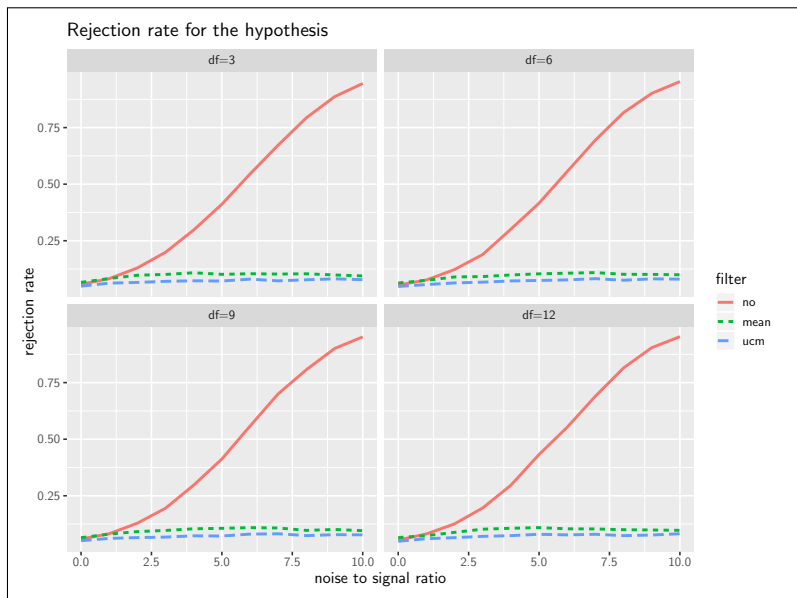
We applied the Johansen test to:

- no** raw data
- mean** aggregated time series by taking the mean every 7 observations (from daily to weekly)
- ucm** smoothed level in a *random walk plus noise* UCM (variances estimated by Gaussian QML)

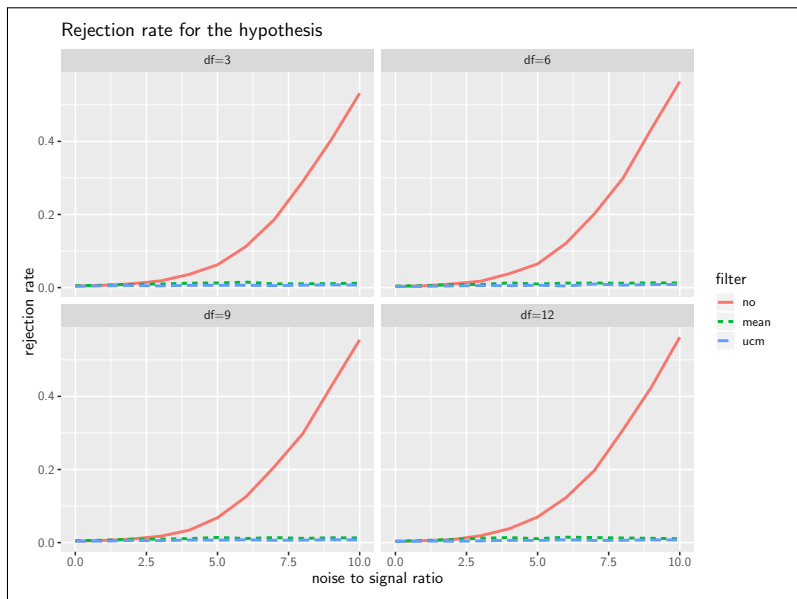
Rejection rates for $H_0 : r = 0$ when $r = 1$



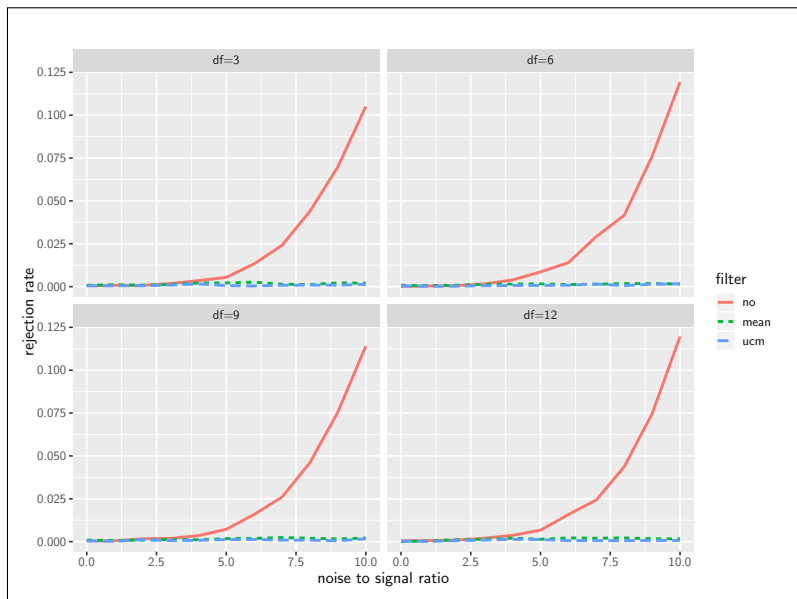
Rejection rates for $H_0 : r = 1$ when $r = 1$



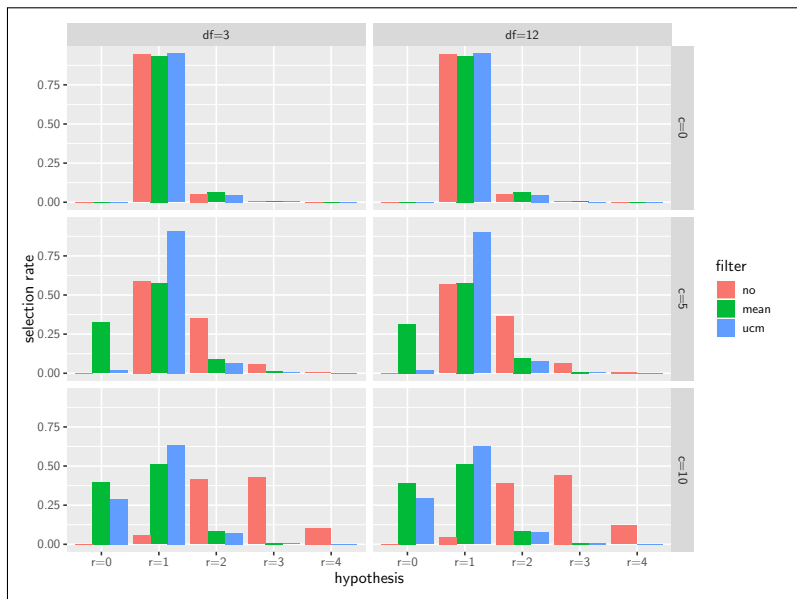
Rejection rates for $H_0 : r = 2$ when $r = 1$



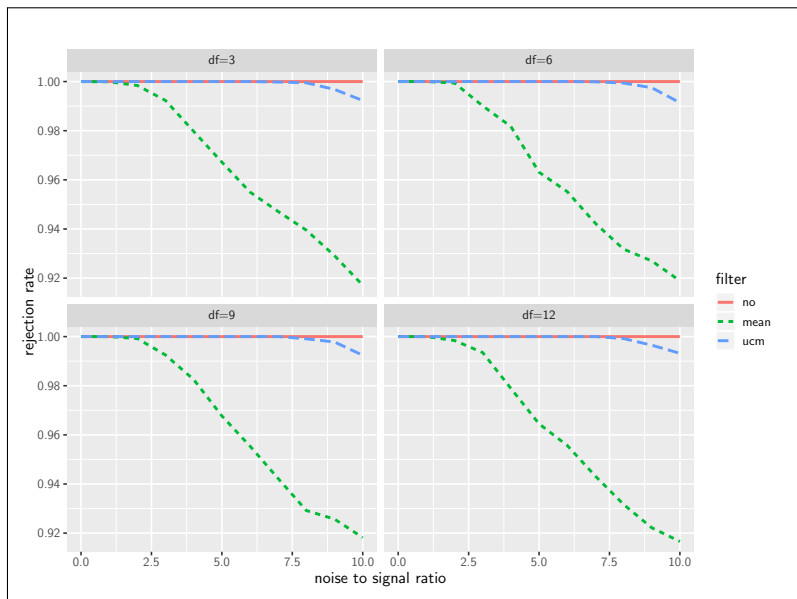
Rejection rates for $H_0 : r = 3$ when $r = 1$



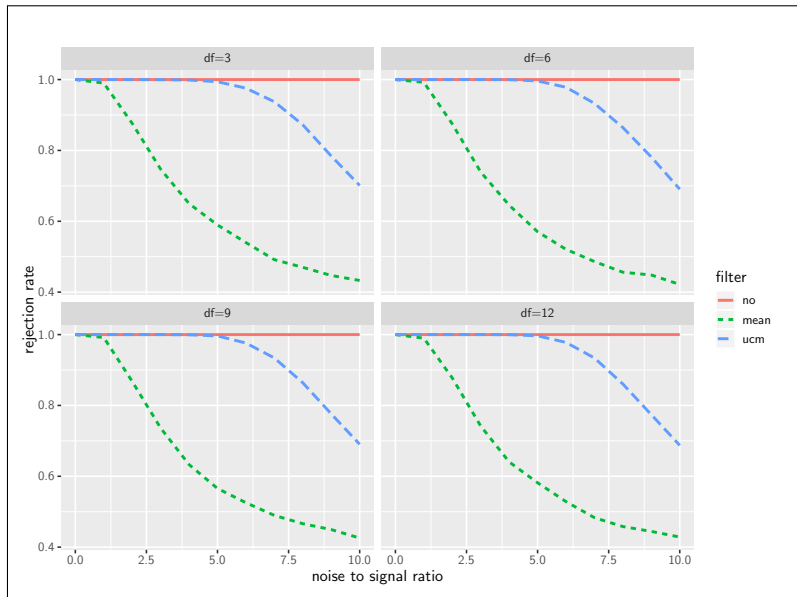
Selection rates when $r = 1$



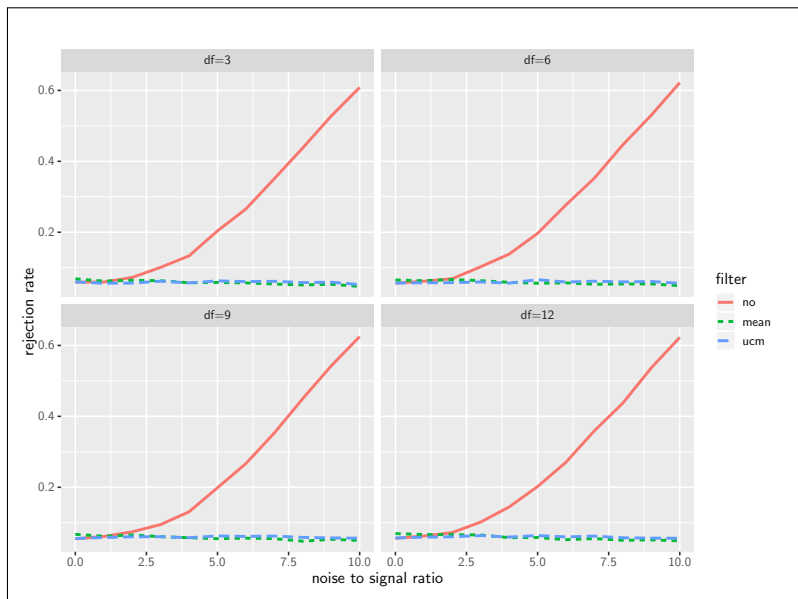
Rejection rates for $H_0 : r = 0$ when $r = 2$



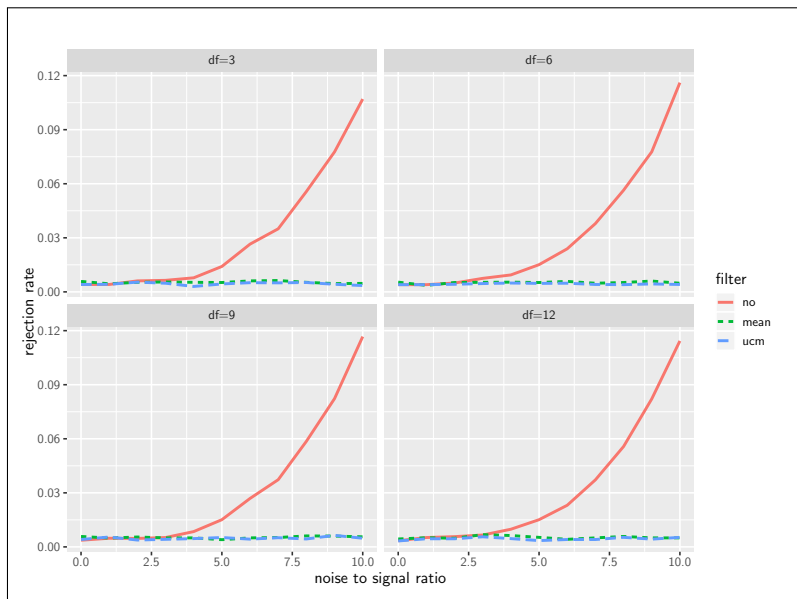
Rejection rates for $H_0 : r = 1$ when $r = 2$



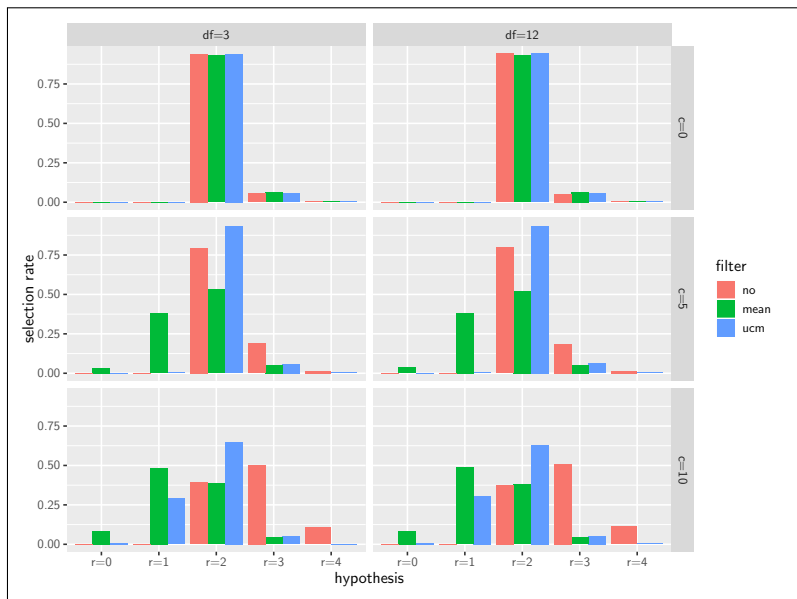
Rejection rates for $H_0 : r = 2$ when $r = 2$



Rejection rates for $H_0 : r = 3$ when $r = 2$



Selection rates when $r = 2$



Conclusions

- ADF and Johansen tests fail when data are very noisy: bias towards **more stationarity**.
- Filtering partially solves, but if there is too much noise bias towards **more integration**.
- Filtering based on **UCM smoothing** seem to work the best.
- Filtering should become routine when looking at long-run features of electricity prices.
- We need to work out theoretical results for UCM-smoothed time series of trend.